Numerical optimization of tokamak and stellarator equilibrium

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Before we begin some background

- Funding levels (2015)
  - 70% W7-X collaboration
  - 20% NSTX-U/Theory partnership
  - 10% LDRD
- Leaving for W7-X for 9? months in March
- Things I’m not talking about today
  - VMEC Benchmarking
  - DIAGNO Benchmarking
  - SPEC ITER/DIII-D Calculations
  - W7-X Field line mapping experiments
Overview of my optimization work

- LHD Equilibrium Reconstruction
  - Initial work with STELLOPT and VMEC
  - Rewrite and validation of magnetic diagnostics code (DIAGNO)
  - MSE synthetic diagnostics into STELLOPT
- Tokamak 3D Equilibrium Reconstruction
  - Rewrite of STELLOPT (STELLOPTV2)
  - DIII-D Reconstruction (NE,TE,TI profile diagnostics)
  - ITER forward modeling of applied RMP’s
- Turbulent Transport Optimization
  - Implementation of parallel GENE in STELLOPTV2
- Development of IPECOPT for NTV optimization on NSTX-U
Optimization of equilibrium in a nutshell.

Input Parameters
- Currents
- Vacuum Fields
- Pressure, etc.

Equilibrium Model

These need to result in a good match to these.

Measurements

Synthetic Signals
Mathematically how can this be achieved?

• Begin with some function we wish to minimize (optimize)

\[ y_j(p_i) : \text{Function} \]
\[ p_i : \text{Parameters} \]

• Given a set of parameters how do we choose better ones?

\[ \tilde{p}_i = p_i + h_i \]
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  \[ \tilde{p}_i = p_i + h_i \]

In practice we minimize

\[ \chi^2(p_i) = \sum_j \left| \frac{Y_j - y_j(p_i)}{\sigma_j} \right|^2 \]
Numerical optimization methods are numerous

• Derivative Methods

  \[ \vec{p}_f = \vec{p}_0 + \vec{h} \]

  • Gradient Descent

    \[ \vec{h}_{GD} = \alpha \vec{J}^\dagger \vec{W} (\vec{y}_\text{target} - \vec{y}(\vec{p})) \]

  • Gauss Newton

    \[ \vec{h}_{GN} = [\vec{J}^\dagger \vec{W} \vec{J}]^{-1} \vec{J}^\dagger \vec{W} (\vec{y}_\text{target} - \vec{y}(\vec{p})) \]

  • Levenberg

    \[ \vec{h}_{LEV} = [\vec{J}^\dagger \vec{W} \vec{J} + \lambda I]^{-1} \vec{J}^\dagger \vec{W} (\vec{y}_\text{target} - \vec{y}(\vec{p})) \]

  • Levenberg-Marquardt

    \[ \vec{h}_{LEV-MAR} = [\vec{J}^\dagger \vec{W} \vec{J} + \lambda \text{diag}(\vec{J}^\dagger \vec{W} \vec{J})]^{-1} \vec{J}^\dagger \vec{W} (\vec{y}_\text{target} - \vec{y}(\vec{p})) \]

• Genetic Algorithms

• Particle Swarm

• Simulated Annealing
The modified Levenberg-Marquardt

- Parallelized over Jacobian evaluation
- Scan over the Levenberg parameter
- STEP_OPT procedure if Jacobian evaluation yields better results than Levenberg step (M. Zarnstorff)
- Failed Jacobian directions removed from the analysis (Robust Jacobian)
Robust Jacobian improves fit

NCSX Reconstruction

Approximately 1/2 of function evaluations forced to fail.
Particle Swarm Optimization (PSO)

- Initial random distribution of points in the parameter hyperspace.
- Each particle has a unique velocity through hyperspace.
- Each particle has a record of its best solution
- All particles keep track of the overall best solution found

Velocity Equation

$$\vec{v}_i^{\text{new}} = \vec{v}_i + C_1 (\vec{x}_i^{\text{best}} - \vec{x}_i) + C_2 (\vec{x}_{\text{global}}^{\text{best}} - \vec{x}_i)$$

Position Update

$$\vec{x}_i^{\text{new}} = \vec{x}_i + \vec{v}_i$$

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STELLOPT fits a VMEC equilibrium to targets

- Optimization Studies
- Global Stability
- Turbulent Transport
- Experimental Design
- HSX, CTH, NCSX
- Reconstruction
- W7-AS, LHD, DIII-D
LHD Equilibrium Reconstruction

- Free boundary 3D equilibrium fit to
  - Thomson
  - Interferrometry
  - MSE
DIII-D 3D Equilibrium Reconstruction

- Free boundary 3D (n=3) equilibrium fit to
  - Magnetics
  - Thomson
  - Charge Exchange
  - Interferrometry
  - MSE
Forward modeling of ITER diagnostic response

- CORSICA used for transport modeling
- STELLOPT used n=3 RMP diagnostic response

Turbulent transport optimization of stellarators

• The capability to optimize stellarators for reduced ITG and TEM turbulence has been demonstrated (Mynick, Pompfrey, Helander, Proll, Xanthopolous)

• The turbulent proxies have been included in STELLOPTV2

• Work is ongoing to incorporate parallel linear GENE into STELLOPTV2

• W7-X should allow validation
An IPEC based optimization code has been developed

- Calculates a least squares fit of IPEC input parameters to target physics parameters
  - Based on STELLOPT
  - Multiple optimization techniques
  - Targeting NTV torque as calculated by PENT
  - Fixed and free boundary optimizations
  - Coil currents can be directly optimized
NSTX-U Core Torque Optimization

- An applied vacuum B-normal spectrum was optimized
  - Core Torque targeting (~0.5 [N.m])
  - n=1 spectrum
RWM+NCC coil optimization

NSTX-U N=1 Core Coil Optimization (NCC+RWM)

- Upper Coils: 192 [A] @ 151°
- RWM Coils: 953 [A] @ 21°
- Lower Coils: 2530 [A] @ 169°
DIII-D error field experiments used to validate

- DIII-D C-coil rotation scan experiments
  - C-coil phase and amplitude scan performed (2D parameter space)
  - Optimizer used for similar experiment (SURFMN error field)
Short term plans (next year)

• Publish IPECOPT results (PPCF)
• Implement 3D equilibrium reconstruction on W7-X
• STELLOPT/GENE runs on Hydra/Hopper
• Energetic particle optimization in STELLOPT
Additional Slides
DIII-D MSE Fit

MSE Polarimetry

Plasma boundary (z=0)

Reconstructed Current Profile

Safety Profile (q)

DIII-D profile comparison (shot 142603)
DIII-D Magnetics

Graphs showing data with axes labeled as B [T], Flux [Wb], Probe Index, and Loop Index.

Graphs with data points and error bars.